A Further Note on Uncertain Gifts and the Gift Economy: Part I

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In an earlier version of this paper (Koda 2009) we have studied an overlapping generations economy populated by two types of agents, type B and type G, where type B agents care only about their own consumption, while type G agents, in addition, care about their parents’ welfare. Each agent’s type is not known until he is born. The setup of the model is otherwise that of Peter Diamond (1965), although agents’ preferences are represented by a quadratic utility function and the production technology is specialized to a linear class. Our aim in this note is to demonstrate that the results therein are robust when the model is extended to a more standard setup, a setup in which each agent’s preferences are represented by a more general class of utility functions and the technology is subject to a standard neoclassical production function.

The remainder of the note is organized as follows. In Section I we describe the economy to be studied. Section II discusses each agent’s decision problem and defines the equilibrium. Section III, before proceeding to the economy populated by both type B and type G agents, studies the two special (polar) cases of our economy, the economy populated by type B agents alone and the economy whose inhabitants are solely of G type. Section III also takes up the issue raised by Abel (1987): Under what conditions each type G agent’s gift motive toward his parent is operative? In the sequel to this paper (Part II) we will address the issue of existence and characterization of a steady state equilibrium of the model where both B type and G type agents coexist.

I. The Model

Consider a discrete time economy with a single good, populated by an infinite sequence of two-period-lived overlapping agents. Each generation is identical in size and contains a continuum of agents with unit mass. Within each generation there are two types of agents, type B and type G, a fraction $p \in (0,1)$ of the population being type B agents. Each agent of generation $t$ has one child at the beginning of date $t+1$. An agent’s type is not known until he is born. Agents of both types care only about old age consumption. While agents of type B care only about their own consumption, agents of type G, in addition, care about their parents’ consumption (or welfare).

Each agent of generation $t$ has a utility function of the form:
\[ U' = \begin{cases} u(c'_{i+1}) & \text{if he is of type B} \\ u(c'_{i+1}) + \gamma U'^{-1} & \text{if he is of type G} \end{cases} \]

where \( U' \) is utility of an agent of generation \( t \), \( c'_{i+1} \) is date \( t+1 \) consumption of an agent of generation \( t \), \( \gamma \) > 0 is the parameter, and \( u(.) \) is such that \( u' > 0, u'' < 0, u'(0) = \infty \), and \( u'(\infty) = 0 \).

Each agent inelastically supplies one unit of labor when young and retires when old. At each date a single good is produced using a neoclassical constant returns to scale production function, which can be written in intensive form as \( f(k) \), where \( k \) is capital per worker and \( f(.) \) satisfies \( f(0) \geq 0, f' > 0, f''(0) = \infty \), and \( f'' < 0 \). We assume that capital stock, after it is used in production, depreciates completely.

II. Each Agent’s Choice Problem and Equilibrium

Each agent works when young and earns the wage income. If he is of type B, then he saves his whole income for his own old age consumption. If he is of type G, then a part of his income is spent to support his parent and the rest is saved for his old age consumption. Since his daughter will behave similarly, his old age consumption will depend on whether she turns out to be of type B or type G. If she happens to be of type B, then his old consumption will consist entirely of his own saved income, or else it will be supported by a gift from his daughter as well.

Before proceeding to agents’ choice problems, we note the following two features of our model. First, following Abel (1987), Junsen and Nishimura (1992, 1993), and O’Conell and Zeldes (1993) and others we adopt the standard Nash assumption that in choosing consumption and gifts each agent takes as given the choices of all other members of his dynastic family.

Second, it should be noted that while type B agents are all alike, type G agents are not despite the fact that they are identical in terms of their labor endowment and their utility function. The reason is that among type G agents their parents’ choices of consumption (and therefore their utility levels), which enter each type G agent’s utility function as an argument, may differ. That way, each agent’s consumption and gift decisions depend on the history of the earlier generations of his dynastic family. Let \( j \) be the number of consecutive generations up to (and including) current one in an agent’s family that were of type G. For example, \( j = 0 \) means that the agent is of type B. If \( j = 1 \), then the agent is of type G but his parent is of type B. If \( j = 2 \), then both the
agent and his parent are of type G but his grandparent was of type B, and so on. In what follows, we will also refer to an agent with family history $j$ as an agent of type $j$ as long as there is no possibility of confusion, so that type 0 is identical to type B and type G is further divided into type $j = 1, 2, \ldots, \infty$, depending on the agent’s family history. Observe that if an agent is of type $j$, then his daughter will be of type $j + 1$ if she happens to be of type G or else she will be of type 0. All agents are indexed by $j$, $j = 0, 1, 2, \ldots, \infty$, and $p(1 - p)^j$ is the fraction of agents who are of type $j$.

We can now state formally each agent’s choice problem. Each agent plays a Bayesian game against his child. Let $q_t(j)$ denote the gift from a type $j$ agent of generation $t$ to his parent. Then each type $j \geq 1$ agent of generation $t$, taking as given his child’s strategy $(q_{t+1}(0), q_{t+1}(j+1))$, chooses his strategy $q_t(j)$ to maximize the expected utility subject to the budget constraints:

$$\max E_t U'(j) = pu(c'_{t+1}(j,0)) + (1 - p)u(c'_{t+1}(j, j+1)) + \gamma U^{t-1}(j - 1)$$

s.t.

$$c'_{t+1}(j,0) = R_{t+1}[w_t - q_t(j)]$$

$$c'_{t+1}(j, j+1) = R_{t+1}[w_t - q_t(j)] + q_{t+1}(j+1)$$

$$c'_{t-1}(j-1, j) = R_t[w_{t-1} - q_{t-1}(j-1)] + q_t(j)$$

where $c'_{t+1}(j,0)$ and $c'_{t+1}(j, j+1)$ are type $j$ agent’s consumption at date $t + 1$ contingent on his child being of type 0 (type B) and type $j + 1$ (type G) respectively.

If, on the other hand, an agent is of type $j = 0$, he in effect faces no choice problem; his consumption plan is completely determined once his child’s strategy is given so that

$$c'_{t+1}(0,0) = R_{t+1}w_t$$

$$c'_{t+1}(0,1) = R_{t+1}w_t + q_{t+1}(1).$$

The first-order condition for the choice problem (1) of a type $j \geq 1$ agent is given by

$$u'[R_t(w_{t-1} - q_{t-1}(j-1)) + q_t(j)] \leq \frac{R_{t+1}}{\gamma} \{pu'[R_{t+1}(w_t - q_t(j))] + (1 - p)u'[R_{t+1}(w_t - q_t(j)) + q_{t+1}(j+1)]\}$$

\[1\] Note that $q_{t+1}(0) \equiv 0$. 

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with equality if \( q_j(j) > 0 \quad \forall j \geq 1 \) \( (2) \)

Perfect competition among firms implies that each factor is paid its marginal product

\[
R_t = R(k_t) \equiv f'(k_t) \quad \text{(3)} \\
w_t = w(k_t) \equiv f(k_t) - k_t f'(k_t). \quad \text{(4)}
\]

Following Boyd and Smith (1998), we assume throughout that, in addition to what we have assumed above, the production function \( f(.) \) satisfies:

**ASSUMPTION 1:**

\[
w'(0) > 1 \quad \text{(5a)}
\]

and

\[
w''(k) < 0 \quad \forall k \geq 0 \quad \text{(5b)}
\]

Finally, if we denote type \( j \geq 0 \) agent’s saving by \( s_i(j) = w_t - q_i(j) \), then a (dynamic) general equilibrium is a set of sequences, \( k_{t+1} > 0 \) and \( q_i(j) \geq 0, \ j = 0,1,2,..., \) such that (2)-(4) as well as the market clearing condition

\[
\sum_{j=0}^{n} p(1-p)^j s_i(j) = k_{t+1}
\]

or equivalently

\[
\sum_{j=0}^{n} p(1-p)^j[w_t - q_i(j)] = k_{t+1}
\]

are satisfied at each date \( t \).

III. Steady State Equilibrium and Operative Gift Motive

From now on we will concentrate on the *steady state* of a general equilibrium of this economy, which we will call a steady state equilibrium or a steady state for short below. The steady state can be characterized by constant distributions of consumption, saving, and gift, and constant values of the wage rate, the gross rate of return, and capital

\footnote{Note that \( w'(k) = -kf''(k) > 0 \quad \forall k > 0 \).}
(investment), which we will denote by \( c(j, 0), c(j, j + 1), \), \( s(j), q(j), w, R, \) and \( k \) respectively. It follows that in the steady state equations (2),(3),(4) and (6) can be replaced by

\[
\begin{align*}
\frac{d}{R} u'[R(w - q(j - 1)) + q(j)] &\leq \frac{R}{\gamma} \{ pu'[R(w - q(j))] + (1 - p)u'[R(w - q(j)) + q(j + 1)] \\
\text{with equality if } q(j) > 0 & \quad \forall j \geq 1
\end{align*}
\] (7)

\[ R = R(k) = f'(k) \] (8)

\[ w = w(k) = f(k) - kf'(k) \] (9)

\[ \sum_{j=0}^{\infty} p(1 - p)^j [w - q(j)] = k \] (10)

A steady state equilibrium is, then, \( k > 0 \) and \( q(j) \geq 0, j = 0, 1, 2, \ldots \), that satisfy (7)-(10). We begin by the following Lemma, which will be essential for the existence, uniqueness, and stability of a steady state equilibrium of our economy.

**LEMMA 1:** Define \( m(k) \equiv w(k) - k \) and let \( k^B > 0 \) be the unique positive solution to \( m(k) = 0 \). Then

\[
\begin{align*}
m(k) &> 0 \text{ if } k \in (0, k^B) \\
m(k) &< 0 \text{ if } k > k^B
\end{align*}
\] (11)

**PROOF:** Define \( \phi(k) \equiv m(k) / k = w(k) / k - 1 \). Then \( \phi'(k) = \frac{1}{k} \left[ w'(k) - \frac{w(k)}{k} \right] < 0 \) for each \( k > 0 \) because \( w(k) \) is strictly concave (Assumption 1). The fact that \( \phi \) is continuous and decreasing, together with the fact that

\[
\lim_{k \to 0} \phi(k) = \lim_{k \to 0} \frac{w(k)}{k} - 1 > 0 \tag{3}
\]

\[
\lim_{k \to \infty} \phi(k) = \lim_{k \to \infty} \frac{w(k)}{k} - 1 = -1 < 0 \tag{4}
\]

implies that there is a unique positive solution, \( k > 0 \), to \( \phi(k) = 0 \). Call it \( k^B \). Then

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3 Either \( w(0) > 0 \) or \( w(0) = 0 \). If the latter holds, use L'Hopital’s rule and Assumption 1 to get the result.

clearly the $k^B$ so defined satisfies (11).

Lemma 1 ensures that there exists a unique (positive) steady state level of capital stock, $k^B$, in the economy without gift motive as studied by Diamond (1965) or equivalently the economy where the entire population consists of type B agents, which we will call the B economy for the sake of brevity. This steady state capital stock is stable and convergence to $k^B$ is monotone (see, for example, Bhattacharyya and Majumdar 2007). To summarize:

PROPOSITION 1: A unique steady state equilibrium exists in the B economy with the level of capital stock $k^B$.

The steady-state rental rate (or the gross rate of return) and wage rate in the B economy are then determined by substituting $k^B$ into (8) and (9) respectively as:

$$R^B = R(k^B) = f'(k^B)$$
$$w^B = w(k^B) = f(k^B) - k^B f'(k^B)$$

where superscript B denotes the B economy.

Before going on, consider another polar-case economy, an economy populated only by type G agents or simply the G economy. To analyze the steady state of this economy we need only set $p = 0$ and $q(j) = q$ for each $j \geq 0$ in (7) and (10):

$$\gamma \leq R \quad \text{with equality if } q > 0$$
$$w - q = k$$

Then the following proposition holds for the G economy.

PROPOSITION 2: A unique steady state equilibrium exists in the G economy such that:

(a) If $\gamma > R^B$, then gifts are positive (operative) in the steady state;
(b) If $\gamma \leq R^B$, then gifts are zero (nonoperative) in the steady state and the steady state of the G economy is identical to that of the B economy.

PROOF: (a) Suppose $\gamma > R^B$. Then (8), (12) and (14) imply that

$$f'(k^G) = R(k^G) = R^G \geq \gamma > R^B = R(k^B) = f'(k^B)$$
so that \( k^G < k^B \). \( q^G = m(k^G) > 0 \) follows from (15) and Lemma 1.

(b) Let \( \gamma \leq R^B \). Suppose by way of contradiction that \( q^G > 0 \). Then (14) implies that \( \gamma = R^G \). However, (15) implies that \( m(k^G) = w(k^G) - k^G = q^G > 0 \) and Lemma 1 in turn implies that \( 0 < k^G < k^B \). This fact, together with (8) and (12), implies that

\[
\gamma = R^G = R(k^G) = f'(k^G) > f'(k^B) = R(k^B) = R^B
\]

which is a contradiction. Once we have \( q^G = 0 \), it is easy from (15) and Lemma 1 to see that \( k^G = k^B \) so that \( R^G = R^B \) and \( w^G = w^B \), from (12) and (13).

Finally, the uniqueness of the steady state follows from the fact that if \( \gamma > R^B \), then \( R^G = \gamma \) and that if \( \gamma \leq R^B \), then the steady state of the G economy is the same as that of the B economy, which is unique.

If agents’ gift motive is sufficiently weak, there will be no transfer from children to parents even in the G economy. Proposition 1, which is essentially a restatement of Andrew Abel’s result (Abel 1987), gives a precise (necessary and sufficient) condition under which gifts are positive in the steady state of the G economy. If the condition in Proposition 1 (a) is met, then the rate of return \( R^G \) will be equal to \( \gamma \), as is implicit in the proof of Proposition 1. Depending on the value of \( \gamma \), the equilibrium in the G economy is dynamically efficient (i.e. if \( \gamma > 1 \)) or inefficient (i.e., if \( \gamma < 1 \)).

It will be useful for our purposes to compare (in terms of \( q, k, R, \) and \( w \)) the steady state of the G economy with operative gifts with that of the B economy, which is equivalent, due to Proposition 1, with comparing the steady states of the G economy with \( \gamma > R^B \) and with \( \gamma \leq R^B \):

\[
q^G > 0 = q^B\\
k^G = f^{-1} (\gamma) < f^{-1} (R^B) = k^B\\
R^G = \gamma = R(k^G) > R(k^B) = R^B\\
w^G = w(k^G) < w(k^B) = w^B.
\]

We are now ready to study the economy populated by both type G and type B agents and to determine the conditions under which the transfer motive from children to parents can be operative in such an economy. Our first step is to rewrite equation (10) as

\[
w(k) - \sum_{j=0}^{\infty} p(1 - p)^j q(j) = k
\]
or

\[ m(k) = w(k) - k = Q \]  \hspace{1cm} (16)

where \( Q \equiv \sum_{j=0}^{\infty} p(1 - p)^j q(j). \) We tentatively assume equations (7)-(9) and (16) have a solution such that \( k > 0 \) and \( q(j) \geq 0, \ \forall j \geq 0 \) so that there is a steady state equilibrium in this economy. Then the following result is immediate.

**Lemma 2:** Suppose that the steady state is such that gifts are positive. Then

(a) \( k < k^B \)
(b) \( R > R^B \)
(c) \( w < w^B \)

**Proof:** (a) Since \( Q > 0, \) (16) implies that \( m(k) > 0. \) Then Lemma 1 implies that \( k < k^B. \) Parts (b) and (c) then follow from the fact that while \( R(k) \) is strictly decreasing in \( k, \) \( w(k) \) is strictly increasing in \( k. \)

We are now in a position to state our main result in this section:

**Proposition 3:**

(a) If \( \gamma > R^B, \) then the steady state is such that gifts are positive.
(b) If \( \gamma \leq R^B, \) then the steady state is identical to that in the B economy.

**Proof:** (a) Let \( \gamma > R^B. \) Suppose by way of contradiction that \( Q = 0 \) so that \( q(j) = 0, \ \forall j \geq 0. \) Then (7) implies that \( R / \gamma \geq 1. \) But if \( Q = 0, \) then (15) and Lemma 1 imply that \( k = k^B, \) so that \( R = R^B (\text{see (8) and (11)).} \) Substitution of this result into \( R / \gamma \geq 1 \) above yields \( \gamma \leq R^B, \) which is a contradiction.

(b) Suppose that \( \gamma \leq R^B \) and suppose by way of contradiction that \( Q > 0 \) so that \( q(j) > 0 \) for some \( j \geq 1. \) Since for such \( j, \) (7) holds at equality, we have

\[
1 = \frac{R}{\gamma} \left\{ p \frac{u[R(w - q(j))]}{u[R(w - q(j) - 1) + q(j)]} + (1 - p) \frac{u[R(w - q(j) + q(j + 1)])}{u[R(w - q(j - 1)) + q(j)]} \right\}
\]

Then the fact that \( u[R(w - q(j))] \geq u[R(w - q(j)) + q(j + 1)] \) implies that

\[
\frac{R u[R(w - q(j)) + q(j + 1)]}{\gamma u[R(w - q(j - 1)) + q(j)]} \leq 1 \hspace{1cm} (17)
\]
Since Lemma 2 (b) and the assumption $\gamma \leq R^B$ imply that $\gamma < R$ or $\gamma / R < 1$, it follows from (17) that

$$\frac{u'[R(w-q(j)) + q(j+1)]}{u'[R(w-q(j-1)) + q(j)]} \leq \frac{\gamma}{R} < 1.$$  

(18)

Since $u'' < 0$, this in turn implies that

$$R(w-q(j)) + q(j+1) > R(q(j-1)) + q(j)$$

or

$$q(j+1) - q(j) > R[q(j) - q(j-1)].$$  

(19)

Let $i = \min\{j \geq 1: q(j) > 0\}$. Then since $q(i) > 0 = q(i-1)$ and (19) holds for $j = i$, we have

$$q(i+1) > q(i) > 0.$$  

By induction (19) holds for each $j \geq i$, which in turn implies that

$$q(j+1) > q(j) > 0$$

for each $j \geq i$; that is, $q(j)$ is strictly increasing in $j$ for each $j \geq i$. But since the sequence $q(j)$ is bounded above by $w$ (see the first term on the right hand side of (7)), it converges to some positive value, say $\bar{q} \in (0, w]$. Now let $j \to \infty$ on both sides of (18) to get

$$\lim_{j \to \infty} \frac{u'[R(w-q(j)) + q(j+1)]}{u'[R(w-q(j-1)) + q(j)]} = \frac{u'[R(w-\bar{q}) + \bar{q}]}{u'[R(w-\bar{q}) + \bar{q}]} = 1 \leq \frac{\gamma}{R} < 1$$

which is a contradiction. This establishes that if $\gamma \leq R^B$, then $Q = 0$ (so that $q(j) = 0$, $\forall j \geq 0$). Finally, once we have $Q = 0$, Lemma 1, (11) and (12) ensure that $k = k^B$, $R = R^B$, and $w = w^B$.

Since our interest is in a steady state in which $\gamma$ is high enough for gifts to be operative (positive), we make the following assumption in the rest of the paper:

ASSUMPTION 2: $\gamma > R^B$.

Then the following holds in the steady state:
LEMMA 3:
(a) $R^G = \gamma > R > R^B$
(b) $k^G < k < k^B$
(c) $w^G < w < w^B$

PROOF: (a) Since the last inequality merely reproduces Lemma 2 (b), it suffices to show the first inequality. To show this, suppose to the contrary that $\gamma \leq R$. Then, repeating what we did in the proof of part (b) of Proposition 2, we are led to exactly the same conclusion: the sequence $q(j)$ converges to some positive value $q^\ast$. (Note that although we need to replace the strict inequalities in (18) and (19) by weak inequalities, we reach the same result: $q(j)$ is strictly increasing in $j$ for each $j \geq i$.) Now letting $j \to \infty$ on both sides of (16) and rearranging yields

$$1 \geq \frac{\gamma}{R} = \left\{1 + p\left[\frac{u'[R(w-q)]}{u'[R(w-q)+\bar{q}]}-1\right]\right\} > 1$$

which is a contradiction.

Parts (b) and (c) follow from part (a) if we note that $k = f^{-1}(R)$ is strictly decreasing in $R$ and that $w = w(k)$ is strictly increasing in $k$.

Given Proposition 3 and Lemma 3, we are now able to show that under Assumption 1 the gift motives for all G agents, $j = 1,2,\ldots$, are operative in the steady state.

PROPOSITION 4: Gifts in the steady state are such that

- $q(j) = 0$ if $j = 0$
- $q(j) > 0$ if $j \geq 1$

PROOF: Suppose by way of contradiction that $q(j) = 0$ for some $j \geq 1$. Then (7) implies

$$u'[R(w-q(j-1))] \leq \frac{R}{\gamma} \left\{pu'(Rw)+(1-p)u'[Rw+q(j+1)]\right\}.$$ 

However, since $q(j-1) \geq 0$ and $q(j+1) \geq 0$, $u'' < 0$ implies that

$$u'[R(w-q(j-1))] \geq u'(Rw) \geq u'[Rw+q(j+1)].$$

Since $R/\gamma < 1$ from Lemma 3, the above inequality cannot hold, a contradiction. This establishes that $q(j) > 0 \ \forall j \geq 1$.

References


